# Developing a New Dry Zenith Path Delay Model for Egypt

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## Abstract

It is commonly known that the natural atmospheric delay is considered one of the major sources of GPS biases that hinder using it in precise geodetic application. The available global dry tropospheric models are derived based on empirical global data. These models are derived using available radiosonde data obtained from Europe and North America continents. The global atmosphere conditions used as constants in these models provide a broad approximation of the tropospheric conditions. Hence, they did not meet the local meteorological conditions of Egypt. In other words, they do not take into account the latitudinal and seasonal variations in the atmosphere**.** Besides, daily variation in temperature, pressure and relative humidity can lead to error in tropospheric delays obtained using the global tropospheric models especially in the height components. Hence these models do not truly reflect the actual tropospheric effect.

With increasing the demands of utilizing GPS in precise geodetic applications, it was inevitably to have a local model to compute the zenith dry tropospheric delay. The current study submits a new local developed model to compute the dry zenith tropospheric delay. The new model was derived from using the ray tracing of meteorological data at 17 sites covering all Egyptian territory. The new developed model was found to be a linear relationship with the measured surface pressure and or measured surface pressure and temperature data. The tested data at the five locations shows a very good performance less than one mm. The results confirm how the new local developed model can enhance the using of GPS in precise geodetic applications.

**Keywords:** GPS, atmospheric delay, global dry tropospheric models, dry zenith tropospheric delay, ray tracing

## 1. Introduction

The troposphere is the lower part of the earth's atmosphere, where the temperature decreases with the altitude, the thickness of the troposphere is not the same everywhere, it extends to a height of less than 9 km over the poles and to more than 16 km over the equator and extends from the sea to about 50 km (**Hofmanen- Wellenhof et al.,2001**). Tropospheric path delay are a major source of error in deep space tracking . However , the tropospheric – induced delay at tracking sites can be calibrated using measurements of Global Positioning System (GPS) satellites. The delay in radio signals caused by troposphere range from 2m at zenith to 20m at lower elevation angles (below 10 degrees) **(Dodo and Kamarudin , 2008**). The tropospheric delay may be divided into two components , dry and wet . Approximately 90% of tropospheric delay caused by refraction is due to dry component of troposphere; it is a function of elevation and altitude of the receiver which depends on factors such as atmospheric temperature, pressure and relative humidity. It is not frequency –dependent as is the case with the ionosphere and cannot be eliminated through linear combination of L1 and L2 observations **(Satirapod and Chalermwattanachai, 2005).** Dry delay may by precisely determine on the strength of the ground meteorological measurements or the model of so called standard atmosphere **(Hugentabler et al., 2001)**. It is because the troposphere accounts for most of the neutral atmosphere mass and contain practically all the water vapor that the term tropospheric delay is often used to designate the effect of the neutral atmosphere **(Hopfield, 1971)**.

## 2. The Aim of the Study

Several global tropospheric models such as the Saastamoinen model, Hopfield model, Niell model etc have been empirically developed and employed in GPS receivers to correct for the tropospheric delay. These models are derived using available radiosonde data obtained from Europe and North America continents. The global atmosphere conditions used as constants in these models, provide a broad approximation of the tropospheric conditions, but ignore the actual atmospheric conditions on a given location. Tthis means that they do not take into account the latitudinal and seasonal variations in the atmosphere (**Roberts and Rizos, 2001).** Besides, daily variation in temperature, pressure and relative humidity can lead to error in tropospheric delays obtained using the global tropospheric models especially in the height components (**Dodo and Kamarudin, 2008**). Hence these models do not truly reflect the actual tropospheric effect. The needs to develop a local model become pertinent.

The aim of this study is to develop a local dry zenith path delay model to meet the accuracy requirements of precise geodetic applications.

## 3. Theoretical

The tropospheric delay consists of the dry and wet components. The dry component is a function of surface pressure and accounts for about 90 % while the wet component is a function of the distribution of water vapour in the atmosphere and represents about 10% of the total delay (**Misra and Enge, 2001)**. Therefore, the tropospheric delay has been shown to be directly proportional to the refractive index and this is functionally expressed as (**Hoffman –Wellenhof et al. 2001**):

$$D^{trop} = \int (n-1)\, ds \tag{1}$$

Expressing in term of refractivity,

$$D^{trop} = 10^{6} \int N^{trop} ds \tag{2}$$

Where:

$N^{trop}$ =$10^{-6}(n-1)$ is the tropospheric refractivity and $n$ is the refractive index.

The delay of GPS signal can be functionally expressed as the sum of dry and wet contributions to the total delay as the flowing:

$$D^{trop} = 10^{6} \int_{path} N_d^{trop} ds + 10^{6} \int_{path} N_w^{trop} ds \tag{3}$$

According to Thayer 1974 expressed the refractivity $N^{trop}$in term of absolute temperature and partial pressure of the dry gases($P_d$)and water vapor $e$) in millibars (**Mendes and Langely, 1994).**

$$N^{trop} = K_1 \frac{P_d}{T} \mathrm{Z_d^{-1}} + k_2' \frac{\mathrm{e}}{\mathrm{T}} \mathrm{Z_w^{-1}} + \mathrm{K_3} \frac{\mathrm{e}}{T^2} \mathrm{Z_w^{-1}} \tag{4}$$

$$= \mathrm{N_d^{trop}} + \mathrm{N_w^{trop}} ,$$

Where:

| | |
|---|---|
| $P_d$ | is the partial pressure of the dry air in mbar |
| $e$ | is the partial pressure of the water vapor in mbar |
| T | is the absolute temperature in Kelvin |
| $\mathrm{Z_d^{-1}}, \mathrm{Z_w^{-1}}$ | are corrections for non- ideal gas behavior for the dry air and water vapor respectively, their values are very close to unity and can be found in (Owens,1967) [Jk$g^{-1}k^{-1}$]. |
| $K_i$ | Empirically determined coefficients (**Mendes, 1999**), [K h P$a^{-1}$] |
| $R_d, R_w$ | Gas constant respectively for dry component and wet component [Jk$g^{-1}k^{-1}$] |

$$k_2' = k_2 - k_1 \frac{R_d}{R_w} \tag{5}$$

Where the constant coefficients $K_1$, $k_2'$ and $K_3$ are empirically determined ,$Z_d$ and $Z_w$ are factor for dry air and water vapor, the frequently used sets of refractivity constant are given in table (1).

Table (1): Frequently used sets of refractivity constants

| **Reference** | $\boldsymbol{K_1}$ **[K hP$\boldsymbol{a^{-1}}$]** | $\boldsymbol{k_2}$**[K hP$\boldsymbol{a^{-1}}$]** | $\boldsymbol{k_3 10^5}$**[K hP$\boldsymbol{a^{-1}}$]** | $\boldsymbol{k_2'}$**[K hP$\boldsymbol{a^{-1}}$]** |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| (Boudouris,1963) | 77.59 ± 0.08 | 72 ± 11 | 3.75 ± 0.03 | 24 ±11 |
| (Smith and Weintraub,1953) | 77.61 ± 0.01 | 72 ± 9 | 3.75 ± 0.03 | 24 ± 9 |
| Thayer,1974) | 77.60 ± 0.01 | 64.79 ± 0.08 | 3.776 ± 0.004 | 17 ± 10 |

.4

The integral of the dry refractivity (dry delay) is difficult to determine from this definition because the dry equation is valid for the total pressure of the dry air only. Fortunately, the first two terms in equation (4) have the same temperature dependence, and using the gas law for each gas "i" with compressibility $Z_i$ , one can get (**Davis et al., 1985**):

$$P_i = \rho_i \frac{R}{m_i} T Z_i \quad (6)$$

Where:

$\rho_i$ is the density
$m_i$ is the molar mass
$R$ is the universal gas constant (8314.34±0.35J/Kmol K),

Equation (4) can be rewritten as:

$$N^{trop} = K_1 \frac{R}{m_d} \rho + K_2' \frac{m_d}{m_w} \frac{e}{T} Z_w^{-1} + K_3 \frac{e}{T^2} Z_w^{-1} \quad (7)$$

Where: $\rho$ is the total density, $m_d$, $m_w$ are the molar masses of dry (28.9644± 0.0014 kg/ kmol) and wet (18.0152 kg/kmol) air, respectively (**Elgered, 1993**].

Since the dry delay $\delta T_d$ in our main concern here, it can be expressed as:

$$\delta T_d = 10^{-6} \int K_1 \frac{R}{m_d} \rho \, ds \quad (8)$$

## 5. Dry Zenith Path Delay

Using equation (8), the dry delay in the zenith direction $\delta T_d^Z$ is obtained by replacing the path elements ds with the vertical elements *dh* in equation (8) as:

$$\delta T_d^Z = 10^{-6} \int K_1 \frac{R}{m_d} \rho \, dh$$

$$= 10^{-6} \int N_d^{trop} \, dh \quad (9)$$

## 6. Data Analysis

- Seventeen point distributed all over Egypt were selected to be used in the design of a new model, as seen in figure (2).

- For every station, the day of year (339), average values of meteorological parameters are used.
- The Mass-Spectrometer –Incoherent –Scatter (MSIS) model, which describes the neutral temperature, densities in earth's atmosphere from ground to thermospheric height (**Hedin, (1991).**, is used to profile of temperature and pressure that is needed for the ray tracing analysis. Ray tracing is the process of determining the path of an electromagnetic signal, based on geometric optics theory applied over a series of thin spherical shells, concentric with the earth, and within which a constant refractivity is assumed.
- Used three different times (at one, eight and thirteen clock) and taking the average values of them).
- Divided the tropospheric layer into different elements as the following :
  (From 0 to 1km) by interval 100 m, (from 1km to 5 km) by interval 200 m and (From 5km to 50 km) by interval 1km as showing the figure (3).

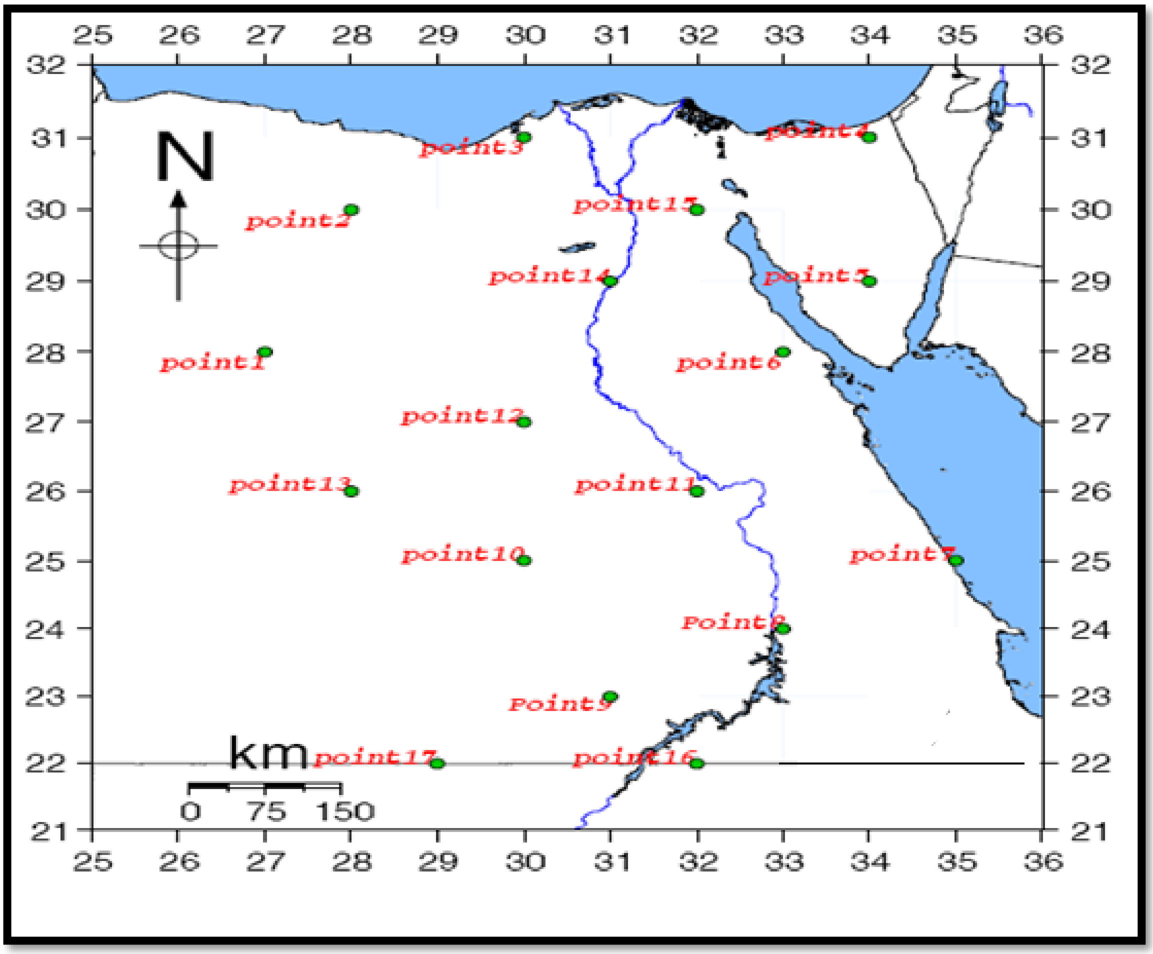


Figure (2): Seventeen point distributed all over Egypt

First, the dry refractivity is calculated from the temperature and pressure values at each layer in three cases (at one, eight and 13 O'clock) using the first part of equation (4), then the ray tracing is carried out numerically by integrating equation (9), the results are given in table (2), figure (4).

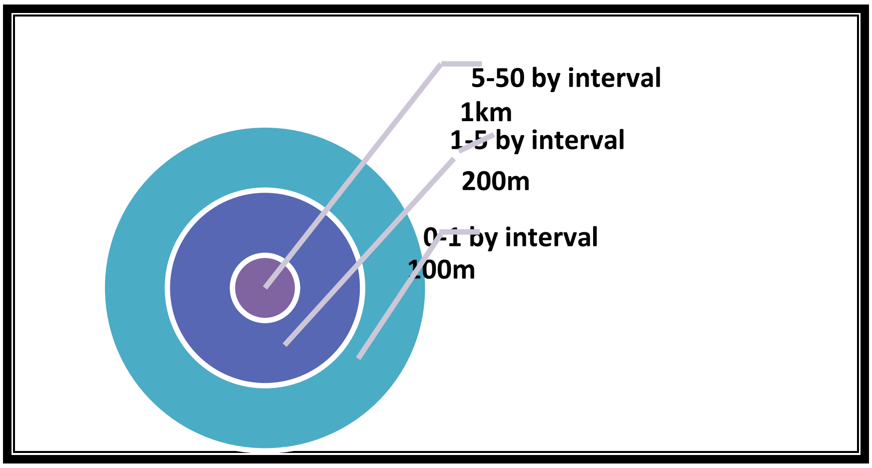


Figure (3): Illustration shows haw the division troposphere height to different elements

Table (2): The Zenith dry delay and surface pressure

| Point | T (K) | P (HP) | $\delta T_d^Z$ (m) | P/T (HP/K) |
|---|---|---|---|---|
| 1 | 294.8 | 1012.958769 | 2.309857756 | 3.436088089 |
| 2 | 293.7 | 1014.237613 | 2.312917773 | 3.453311588 |
| 3 | 293 | 1014.343538 | 2.314309003 | 3.461923338 |
| 4 | 292.9 | 1014.838139 | 2.314001264 | 3.464793921 |
| 5 | 294.1 | 1013.086222 | 2.310920197 | 3.444699838 |
| 6 | 294.7 | 1012.61516 | 2.309310887 | 3.436088089 |
| 7 | 296.3 | 1010.457917 | 2.304232913 | 3.41025284 |
| 8 | 296.8 | 1009.607076 | 2.302786327 | 3.40164109 |
| 9 | 297.3 | 1008.747623 | 2.301515385 | 3.393029341 |
| 10 | 296.3 | 1010.457917 | 2.304688006 | 3.41025284 |
| 11 | 295.8 | 1011.300146 | 2.306134814 | 3.41886459 |
| 12 | 295.3 | 1012.133763 | 2.307990716 | 3.427476339 |
| 13 | 295.9 | 1011.642032 | 2.306484766 | 3.41886459 |
| 14 | 294.2 | 1013.430692 | 2.311212232 | 3.444699838 |
| 15 | 293.6 | 1013.892282 | 2.312689224 | 3.453311588 |
| 16 | 297.7 | 1008.39569 | 2.29982321 | 3.387288174 |
| 17 | 297.7 | 1008.39569 | 2.300157569 | 3.387288174 |

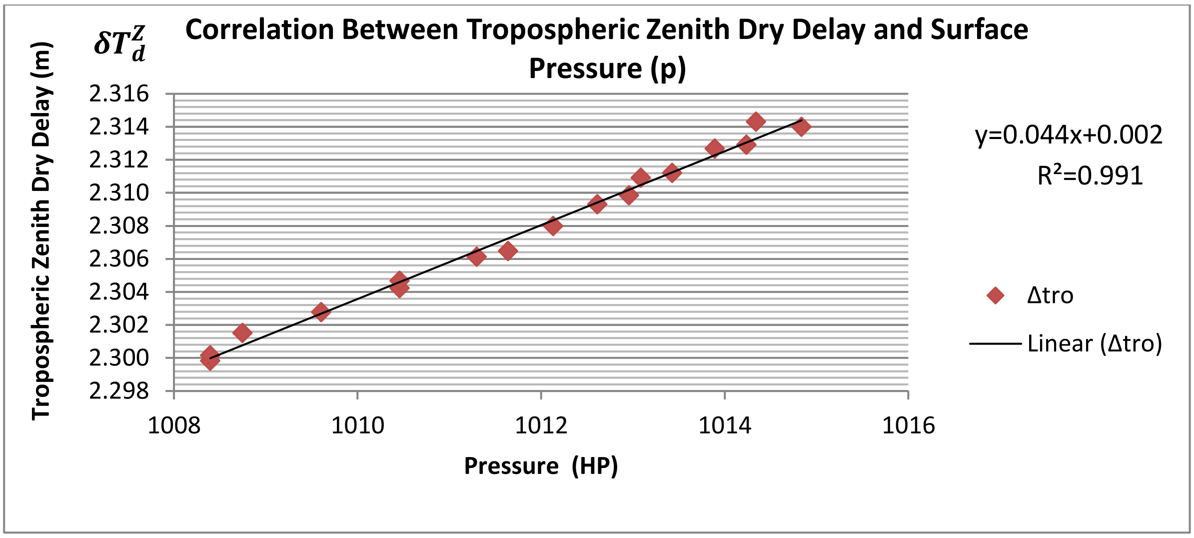


Figure (4): Correlation between Zenith dry delay and surface pressure

Figure (4) clearly indicates that the ray traced zenith delay show the correlation with surface pressure and zenith path delay, the correlation coefficient was found to be 0.991. Thus a linear relation exists between the dry delay and the surface pressure. Therefore, the new local model takes the form:

$$\delta T_d^Z = a * P + b \tag{10}$$

**With: *a* = 0.002, *b* = 0.044**

Also the refractive at any point in a dry atmosphere depends on the pressure and temperature (the ratio P/T in Equation (4). Thus a linear relation between zenith dry delay with the surface pressure and temperature figure (5).

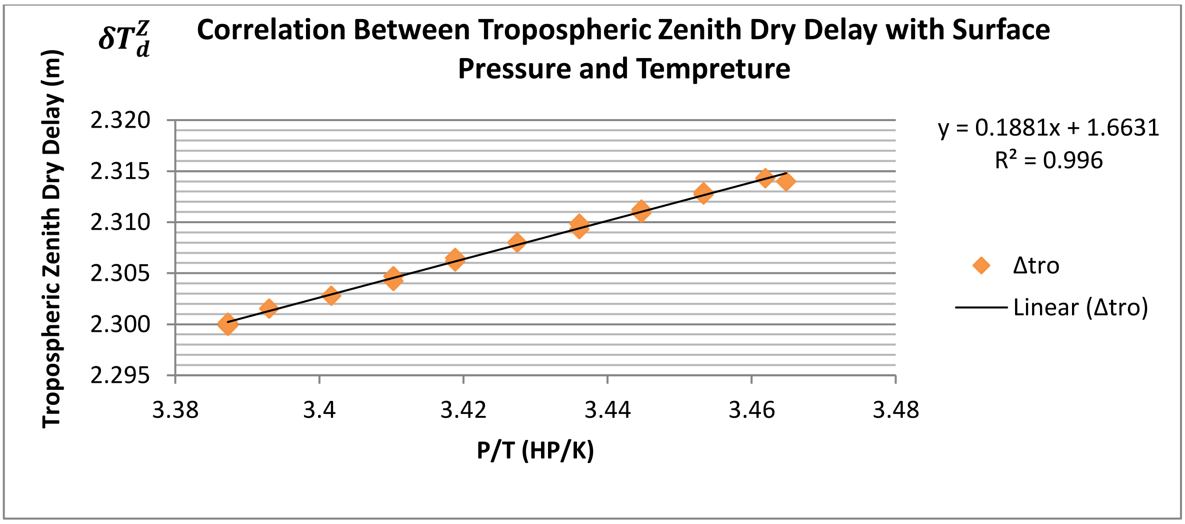


Figure (5): The linear relationship between Zenith dry delay with surface pressure and temperature

Figure (5) clearly indicates that shows the linearity between the ray traced zenith delay with surface pressure and temperature are very strong compared to the stand alone surface pressure as depicted in figure (4). The correlation coefficient was found to be 0.996. Thus a linear relation exists between the dry delay, surface pressure and temperature. Therefore, the second new local model takes the form:

$$\delta T_d^Z = a * \frac{P}{T} + b \qquad (11)$$

**With *a* = 0.188 and *b* = 1.663** as constant parameters.

## 7. Verifying the Developed Model

To verify the developed model, A performance evaluation of the new model was done against two of the precise global tropospheric models, namely Hopfield, Saastamoinen and one local model namely, Mousa et al.,(2003) model (Mousa et al, 2003), derived a local model based on simulated surface pressure. Figure (6) clearly indicates that the ray traced zenith delays show very strong correlation with simulated surface pressure. The correlation coefficient was found to be 0.9995. Thus a linear relation exists between the dry delay and surface pressure. Therefore, the new model for the Egyptian meteorological conditions takes the form:

$$\Delta L^Z{}_h = a + b * P, \qquad (12)$$

With: *a* and *b* as constant parameters. These two constant are estimated from the data fitting to be a = 35.482674 (± 0.00009) mm and b = 2.2460948 (± 0.008) mm/mbar, respectively.

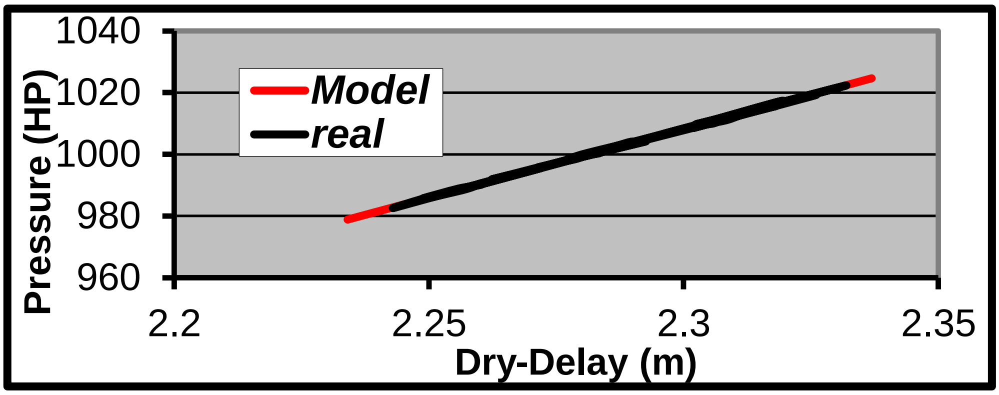


Figure (6): Correlation between Zenith dry delay and surface pressure (Mousa et al, 2003)

The verification was performed over five points distributed over Egypt. The five test locations are Salum, Helwan, Safaga, Alis (Aswan) and Shlatin. The location of the five test locations are illustrated in figure (7). For every test location, the dry zenith tropospheric ray tracing is computed, as described above, as well as by the new model and the two global and one local tropospheric models. The computed values are tabulated in table (4) and the differences were drawn in figure (8).

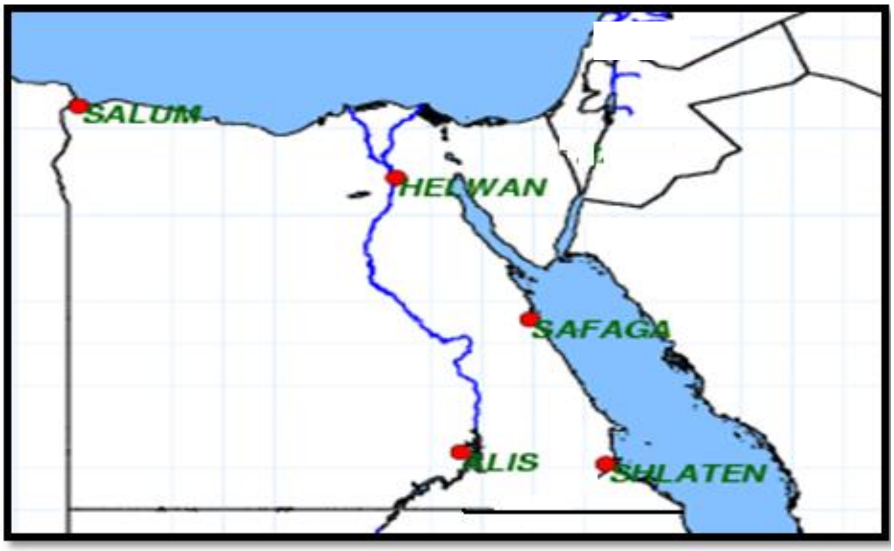


Figure (7): Represents five points to check the error of estimated model

Table (4): The performance of developed new model against Global, Local models

| Testing points | Total Dry Zenith delay (m) | Hopfield | Saastamoinen | Mousa et al., 2003 | New model |
|---|---|---|---|---|---|
| | | **Difference(m)** | **Difference(m)** | **Difference(m)** | **Difference(m)** |
| **SALUM** | **2.315402** | 2.319236 | 2.305592 | 2.31679377 | 2.31534 |
| | | **0.00383** | **-0.00981** | **0.00139** | **-0.00006** |
| **HELWAN** | **2.312399** | 2.316016 | 2.3023194 | 2.31355649 | 2.3126400 |
| | | **0.00362** | **-0.01008** | **0.00157** | **-0.00024** |
| **SAFG** | **2.306914** | 2.309063 | 2.295249 | 2.30656011 | 2.306701 |
| | | **0.00215** | **-0.01166** | **0.00035** | **-0.00021** |
| **SHLATIN** | **2.3013241** | 2.303012 | 2.289087 | 2.30046331 | 2.301301 |
| | | **0.00169** | **-0.0122** | **0.00086** | **-0.00002** |
| **ALIS** | **2.3018634** | 2.304176 | 2.2902527 | 2.30161691 | 2.301841 |
| | | **0.00231** | **-0.0116** | **0.00025** | **-0.00002** |

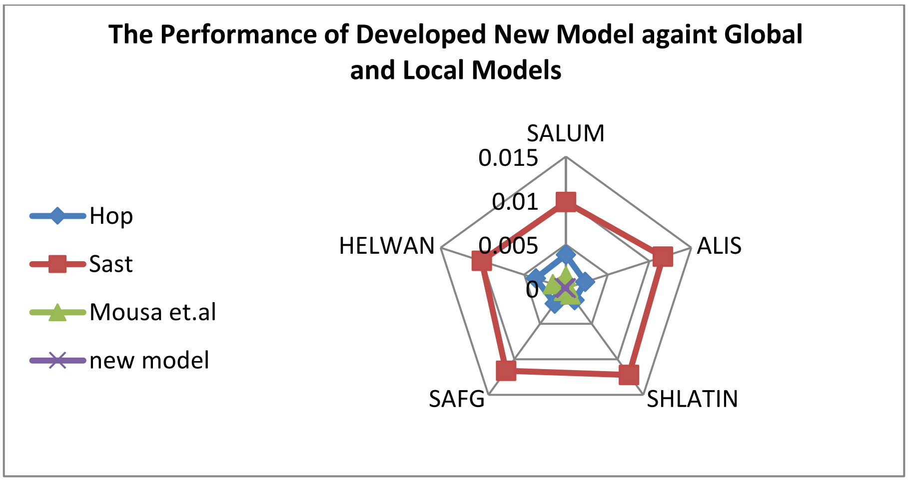


Figure (8): The performance of developed new model against Global and Local Models

From the differences in table (4), new model yield less differences compared with another models.

## Conclusions

As a closing remark for this part, it is commonly known that the natural atmospheric delay is considered one of the major sources of GPS biases that hinder using it in precise geodetic application. The available dry tropospheric models were derived based on empirical global data which did not meet the local meteorological conditions of Egypt. With increasing the demands of utilizing GPS in precise geodetic applications, it was inevitably to have a local model to compute the zenith dry tropospheric delay. The current study submits a new local developed model to compute the dry tropospheric delay. The new model was derived from using the ray tracing of meteorological data at 17 sites covering all Egyptian territory. The new developed model was found to be a linear relationship with the measured surface pressure and or measured surface pressure and temperature data. The tested data at the five locations shows a very good performance less than one mm. The results confirm how the new local developed model can enhance the using of GPS in precise geodetic applications.

## References